Exercise 26

Find the domain of the function $f(x) = \sqrt{2x^3 - 50x}$ by:

(a) using algebra.

(b) graphing the function in the radicand and determining intervals on the x-axis for which the radicand is nonnegative.

Solution

You cannot take the square root of a negative number, so it's necessary that

$$2x^3 - 50x \geq 0$$

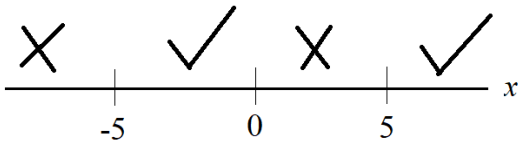
Factor the left side.

$$2x(x^2 - 25) \geq 0$$

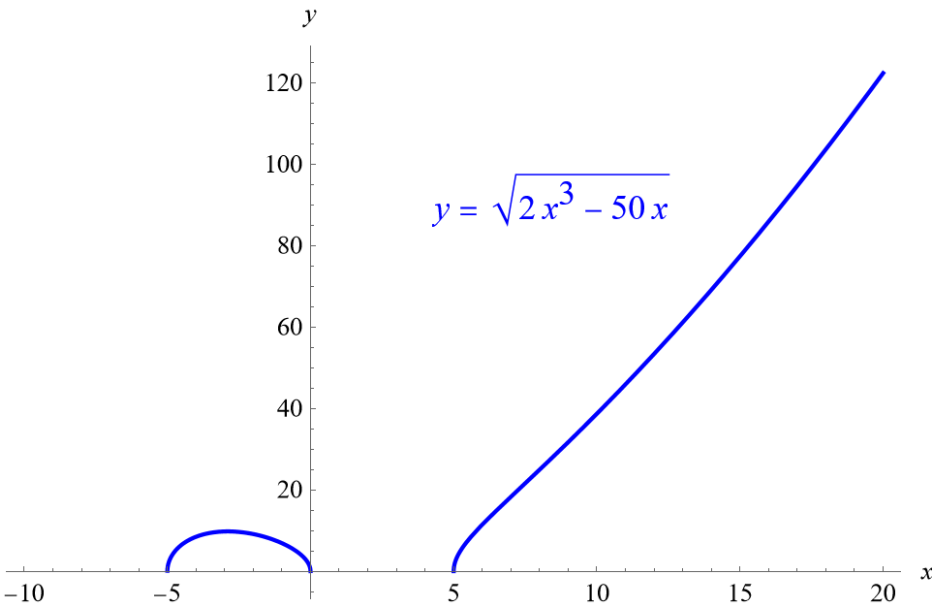
Factor the left side further.

$$2x(x + 5)(x - 5) \geq 0$$

The critical points are $x = \{-5, 0, 5\}$. Divide the number line at these values of x and test where the inequality is true.



Therefore, the domain is $[-5, 0] \cup [5, \infty)$. This is reflected in the graph of $f(x)$ versus x.



This is also where the radicand is nonnegative.